ROTATIONAL MOTION IN NUCLEI

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by

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The exploration of nuclear structure over the last quarter century has been a rich experience for those who have had the privilege to participate. As the nucleus has been subjected to more and more penetrating probes, it has continued to reveal unexpected facets and to open new perspectives. The preparation of our talks today has been an occasion for Ben Mottelson and myself to relive the excitement of this period and to recall the interplay of so many ideas and discoveries coming from the worldwide community of nuclear physicists, as well as the warmth of the personal relations that have been involved.

In this development, the study of rotational motion has had a special role. Because of the simplicity of this mode of excitation and the many quantitative relations it implies, it has been an important testing ground for many of the general ideas on nuclear dynamics. Indeed, the response to rotational motion has played a prominent role in the development of dynamical concepts ranging from celestial mechanics to the spectra of elementary particles.

EARLY IDEAS ON NUCLEAR ROTATION

The question of whether nuclei can rotate became an issue already in the very early days of nuclear spectroscopy (1, 2). Quantized rotational motion had been encountered in molecular spectra (3), but atoms provide examples of quantal systems that do not rotate collectively. The available data on nuclear excitation spectra, as obtained for example from the fine structure of a decay, appeared to provide evidence against the occurrence of low-lying rotational excitations, but the discussion was hampered by the expectation that rotational motion would either be a property of all nuclei or be generally excluded, as in atoms, and by the assumption that the moment of inertia would have the rigid-body value, as in molecular rotations. The issue, however, took a totally new form with the establishment of the nuclear shell model (4).

Just at that time, in early 1949, I came to Columbia University as a research fellow and had the good fortune of working in the stimulating atmosphere of the Pupin Laboratory where so many great discoveries were being made under the inspiring leadership of I.I. Rabi. One of the areas of great activity was the study of nuclear moments, which was playing such a crucial role in the development of the new ideas on nuclear structure.

To-day, it is difficult to fully imagine the great impact of the evidence for nuclear shell structure on the physicists brought up with the concepts of the liquid-drop and compound-nucleus models, which had provided the basis for interpreting nuclear phenomena over the previous decade (5)[1]. I would like also to recall my father's reaction to the new evidence, which presented the sort of dilemma that he would respond to as a welcome opportunity for deeper understanding. In the summer of 1949, he was in contact with John Wheeler on the continuation of their work on the fission process, and in this connection, in order to "clear his thoughts", he wrote some tentative comments on the incorporation of the contrasting evidence into a more general picture of nuclear constitution and the implications for nuclear reactions (7). These comments helped to stimulate my own thinking on the subject, which was primarily concerned with the interpretation of nuclear moments[2].

The evidence on magnetic moments, which at the time constituted one of the most extensive quantitative bodies of data on nuclear properties, presented a special challenge. The moments showed a striking correlation with the predictions of the one-particle model (9, 4), but at the same time exhibited major deviations indicative of an important missing element. The incomparable precision that had been achieved in the determination of the magnetic moments, as well as in the measurement of the hyperfine structure following the pioneering work of Rabi, Bloch, and Purcell, was even able to provide information on the distribution of magnetism inside the nucleus (10, 11).

A clue for understanding the deviations in the nuclear coupling scheme from that of the single-particle model was provided by the fact that many nuclei have quadrupole moments that are more than an order of magnitude larger than could be attributed to a single particle[3]. This finding directly implied a sharing of angular momentum with many particles, and might seem to imply a break-down of the one-particle model. However, essential features of the single-particle model could be retained by assuming that the average nuclear field in which a nucleon moves deviates from spherical symmetry (15). This picture leads to a nuclear model resembling that of a molecule, in which the nuclear core possesses vibrational and rotational degrees

[1] The struggle involved in facing up to the new evidence is vividly described by Jensen (6). Our discussions with Hans Jensen over the years concerning many of the crucial issues in the development provided for us a special challenge and inspiration.

[2] The interplay between individual-particle and collective motion was also at that time taken up by John Wheeler. Together with David Hill, he later published the extensive article on "Nuclear Constitution and the Interpretation of Fission Phenomena" (8), which has continued over the years to provide inspiration for the understanding of new features of nuclear phenomena.

[3] The first evidence for a non-spherical nuclear shape came from the observation of a quadrupole component in the hyperfine structure of optical spectra (12). The analysis showed that the electric quadrupole moments of the nuclei concerned were more than an order of magnitude greater than the maximum value that could be attributed to a single proton and suggested a deformation of the nucleus as a whole (13). The problem of the large quadrupole moments came into focus with the rapid accumulation of evidence on nuclear quadrupole moments in the years after the war and the analysis of these moments on the basis of the shell model (14).

of freedom. For the rotational motion there seemed no reason to expect the classical rigid-body value; however, the large number of nucleons participating in the deformation suggested that the rotational frequency would be small compared with those associated with the motion of the individual particles. In such a situation, one obtains definite limiting coupling schemes (see Fig. 1) which could be compared with the empirical magnetic moments and the evidence on the distribution of nuclear magnetism, with encouraging results (15, 17)[4].

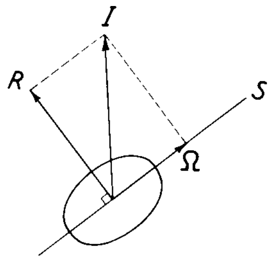


Fig. 1. Coupling scheme for particle in slowly rotating spheroidal nucleus. The intrinsic quantum number represents the projection of the particle angular momentum along the nuclear symmetry axis S, while ***R*** is the collective angular momentum of the nuclear core and is directed perpendicular to the symmetry axis, since the component along S which is a constant of the motion, vanishes in the nuclear ground state. The total angular momentum is denoted by I. The figure is from (16).

In the meantime and, in fact, at nearly the same point in space, James Rainwater had been thinking about the origin of the large nuclear quadrupole moments and conceived an idea that was to play a crucial role in the following development. He realized that a non-spherical equilibrium shape would arise as a direct consequence of single-particle motion in anisotropic orbits, when one takes into account the deformability of the nucleus as a whole, as in the liquid-drop model (19).

On my return to Copenhagen in the autumn of 1950, I took up the problem of incorporating the coupling suggested by Rainwater into a consistent dynamical system describing the motion of a particle in a deformable core. For this coupled system, the rotational motion emerges as a low-frequency component of the vibrational degrees of freedom, for sufficiently strong coupling. The rotational motion resembles a wave travelling across the nuclear surface and the moment of inertia is much smaller than for rigid rotation (see Fig. 2).

Soon, I was joined by Ben Mottelson in pursuing the consequences of the interplay of individual-particle and collective motion for the great variety of nuclear phenomena that was then coming within the range of experimental

[4] The effect on the magnetic moments of a sharing of angular momentum between the single particle and oscillations of the nuclear surface was considered at the same time by Foldy and Milford (18).

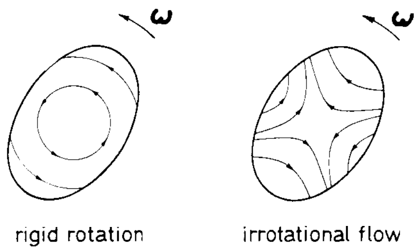


Fig. 2. Velocity fields for rotational motion. For the rotation generated by irrotational flow, the velocity is proportional to the nuclear deformation (amplitude of the travelling wave). Thus, for a spheroidal shape, the moment of inertia is $\mathcal{J} = \mathcal{J}_{\mathrm{rig}}(\Delta R/R)^2$, where $\mathcal{J}_{\mathrm{rig}}$ is the moment for rigid rotation, while $\boldsymbol{R}$ is the mean radius and $\Delta \boldsymbol{R}$ (assumed small compared with $\boldsymbol{R}$) is the difference between major and minor semi-axes. The figure is from (16).

studies (20). In addition to the nuclear moments, important new evidence had come from the classification of the nuclear isomers (21) and beta decay (22) as well as from the discovery of single-particle motion in nuclear reactions (23, 24). It appeared that one had a framework for bringing together most of the available evidence, but in the quantitative confrontation with experiment, one faced the uncertainty in the parameters describing the collective properties of the nucleus. It was already clear that the liquid-drop description was inadequate, and one lacked a basis for evaluating the effect of the shell structure on the collective parameters.

THE DISCOVERY OF ROTATIONAL SPECTRA

At this point, one obtained a foothold through the discovery that the coupling scheme characteristic of strongly deformed nuclei with the striking rotational band structure was in fact realized for an extensive class of nuclei. The first indication had come from the realization by Goldhaber and Sunyar that the electric quadrupole transition rates for the decay of low-lying excited states in even-even nuclei were, in some cases, much greater than could be accounted for by a single-particle transition and thus suggested a collective mode of excitation (21) . A rotational interpretation (25) yielded values for the nuclear eccentricity in promising agreement with those deduced from the spectroscopic quadrupole moments.

Soon after, the evidence began to accumulate that these excitations were part of a level sequence with angular momenta I = 0, 2, 4 . . . and energies proportional to $I(I+1)$ (26, 27); examples of the first such spectra are shown in Fig. 3. For ourselves, it was a thrilling experience to receive a prepublication copy of the 1953 compilation by Hollander, Perlman, and Seaborg (29) with its wealth of information on radioactive transitions, which made it possible to identify so many rotational sequences.

The exciting spring of 1953 culminated with the discovery of the Coulomb excitation process (30, 31) which opened the possibility for a systematic study of rotational excitations (30, 32). Already the very first experiments by

Huus and Zupančič (see Fig. 4) provided a decisive quantitative test of the rotational coupling scheme in an odd nucleus, involving the strong coupling between intrinsic and rotational angular momenta[5].

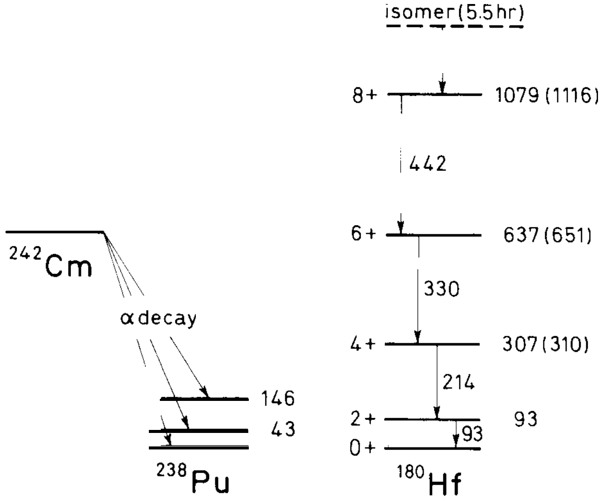


Fig. 3. Rotational spectra for ^{238}Pu and ^{180}Hf. The spectrum of ^{180}Hf (from (26)) was deduced from the observed γ lines associated with the decay of the isomeric state (28). The energies are in keV, and the numbers in parenthesis are calculated from the energy of the first excited state, assuming the energies to be proportional to $I(I+1)$.

The spectrum of ^{238}Pu was established by Asaro and Perlman (27) from measurements of the fine structure in the a decay of ^{242}Cm. Subsequent evidence showed the spin-parity sequence to be 0+, 2+ , 4+, and the energies are seen to be closely proportional to $I(I+1)$.

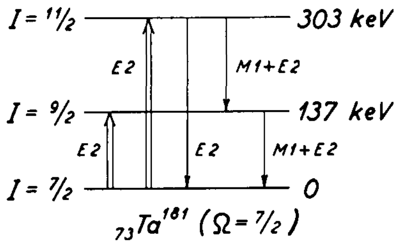


Fig. 4. Rotational excitations in ^{181}Ta observed by Coulomb excitation. In an odd-*A* nucleus with intrinsic angular momentum Ω (see Fig. 1), the rotational excitationsinvolve thesequence $I = \Omega, \Omega+1, \Omega+2, \ldots$, all with the same parity. In the Coulomb excitation process, the action of the electric field of the projectile on the nuclear quadrupole moment induces E2 (electric quadrupole) transitions and can thus populate the first two rotational excitations. The observed energies (30) are seen to be approximately proportional to $I(I+1)$.

The excited states decay by E2 and Ml (magnetic dipole) transitions, and the rotational interpretation implies simple intensity relations. For example, the reduced E2 matrix elements within the band are proportional to the Clebsch-Gordan coefficient $\langle I_i\Omega\, 20 | I_f\Omega \rangle$, where I_i and I_f are the angular momenta of initial and final states. The figure is from (16).

[5] The quantitative interpretation of the cross sections could be based on the semi-classical theory of Coulomb excitation developed by Ter-Martirosyan (33) and Alder and Winther (34).

This was a period of almost explosive development in the power and versatility of nuclear spectroscopy, which rapidly led to a very extensive body of data on nuclear rotational spectra. The development went hand in hand with a clarification and expansion of the theoretical basis.

Fig. 5 shows the region of nuclei in which rotational band structure has so far been identified. The vertical and horizontal lines indicate neutron and proton numbers that form closed shells, and the strongly deformed nuclei are seen to occur in regions where there are many particles in unfilled shells that can contribute to the deformation.

The rotational coupling scheme could be tested not only by the sequence of spin values and regularities in the energy separations, but also by the intensity relations that govern transitions leading to different members of a rotational band (37, 38, 39). The leading order intensity rules are of a purely geometrical character depending only on the rotational quantum numbers and the multipolarity of the transitions (see the examples in Fig. 4 and Fig. 10).

The basis for the rotational coupling scheme and its predictive power were greatly strengthened by the recognition that the low-lying bands in odd-A nuclei could be associated with one-particle orbits in the deformed potential (40, 41, 42). The example in Fig. 6 shows the spectrum of ^{235}U with its high level density and apparently great complexity. However, as indicated, the states can be grouped into rotational bands that correspond uniquely to those expected from the Nilsson diagram shown in Fig. 7.

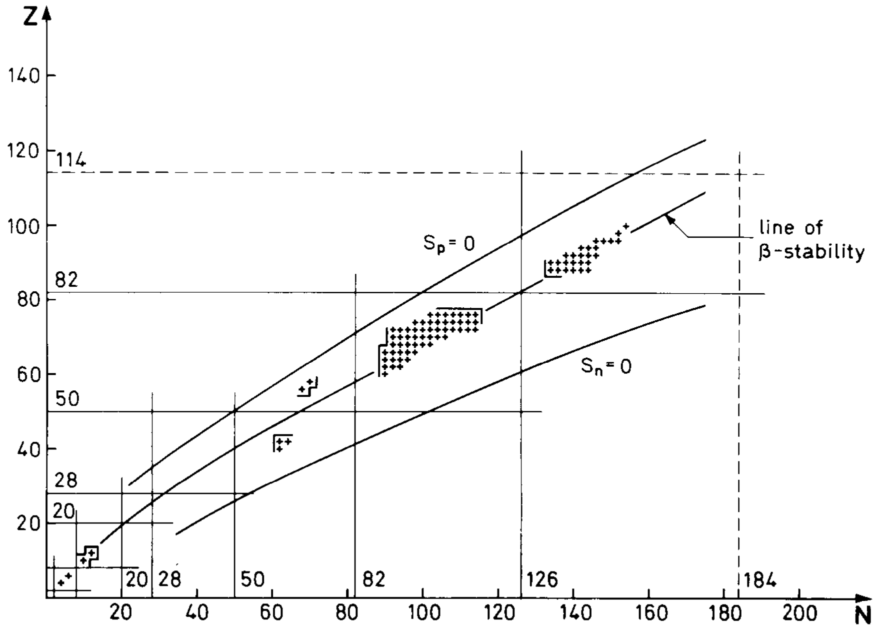


Fig. 5. Regions of deformed nuclei. The crosses represent even-even nuclei, whose excitation spectra exhibit an approximate $I(I+1)$ dependence, indicating rotational band structure. The figure is from (35) and is based on the data in (36). The curves labelled $S_n = 0$ and $S_p = 0$ are the estimated borders of instability with respect to neutron and proton emission.

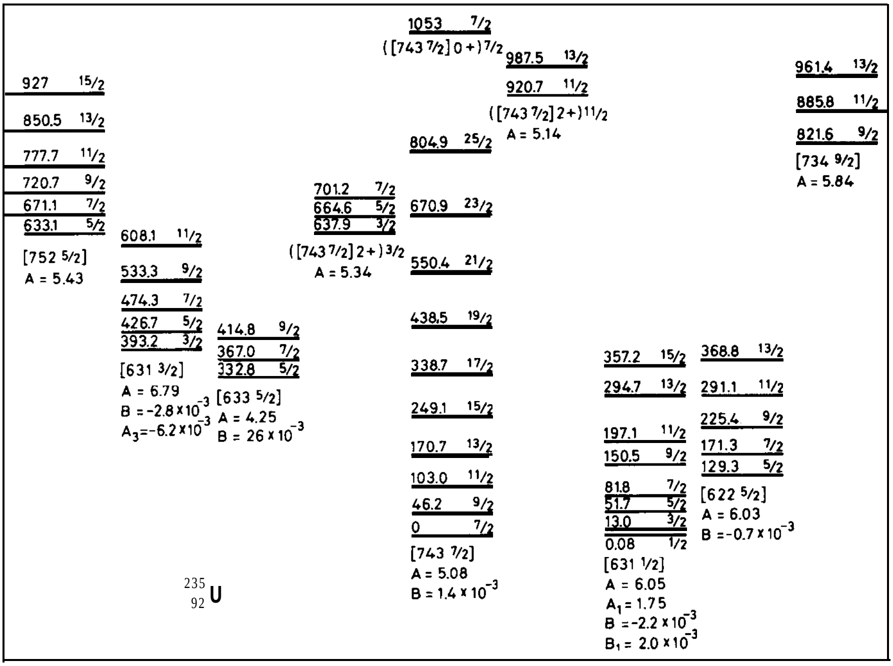


Fig. 6. Spectrum of ^{235}U. The figure is from (35) and is based on the experimental data from Coulomb excitation (43), ^{239}Pu α decay (43a), one-particle transfer (44), and the ^{234}U (n γ) reaction (45). All energies are in keV. The levels are grouped into rotational bands characterized by the spin sequence, energy dependence, and intensity rules. The energies within a band can be represented by a power series expansion of the form $E(I) = AI(I+1) + BI^2(I+1)^2 + \ldots (-1)^{I+\Omega}(I+\Omega)!((I-\Omega)!)^{-1}(A_{2\Omega}+B_{2\Omega}\, I(I+1) + \ldots)$, with the parameters given in the figure. The low-lying bands are labelled by the quantum numbers of the available single-particle orbits (see Fig. 7), with particle-like states drawn to the right of the ground-state band and hole-like states to the left. The bands beginning at 638, 921, and 1053 keV represent quadrupole vibrational excitations of the ground-state configuration.

The regions of deformation in Fig. 5 refer to the nuclear ground-state configurations; another dimension is associated with the possibility of excited states with equilibrium shapes quite different from those of the ground state. For example, some of the closed-shell nuclei are found to have strongly deformed excited configurations[6]. Another example of sharpe isomerism with associated rotational band structure is encountered in the metastable, very strongly deformed states that occur in heavy nuclei along the path to fission (50, 51).

6 The fact that the first excited states in ^{16}O and ^{40}Ca have positive parity, while the low-lying single-particle excitations are restricted to negative parity, implies that these states involve the excitation of a larger number of particles. It was suggested (47) that the excited positive parity states might be associated with collective quadrupole deformations. The existence of a rotational band structure in ^{16}O was convincingly established as a result of the ^{12}C (*aa*) studies (48) and the observation of strongly enhanced E2-transition matrix elements (49).

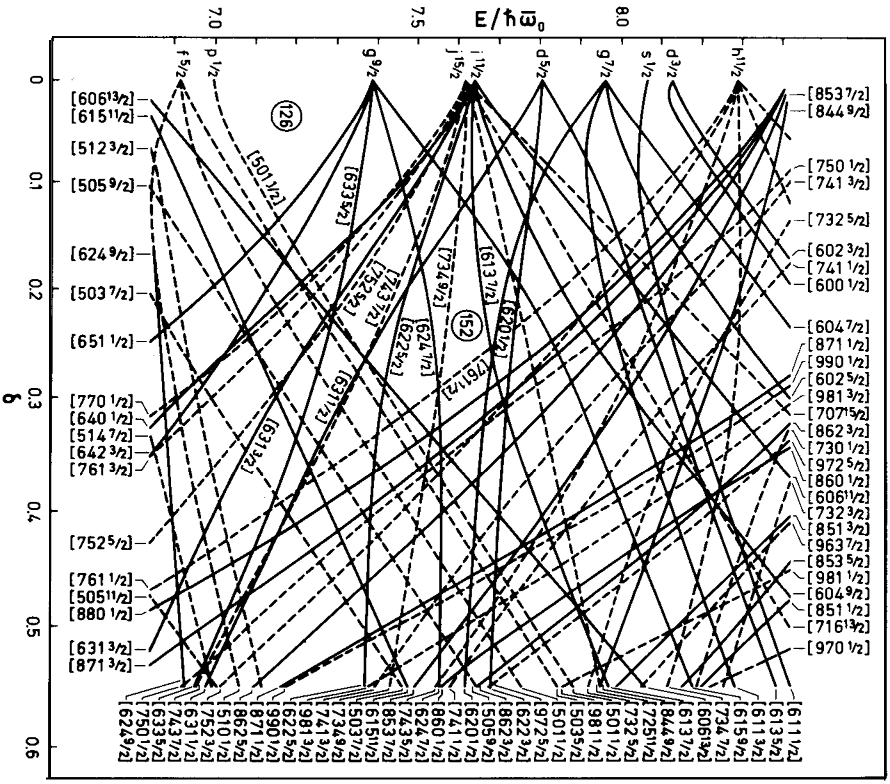


Fig. 7. Neutron orbits in prolate potential. The figure (from (35)) shows the energies of single-particle orbits calculated in an appropriate nuclear potential by Gustafson, Lamm, Nilsson, and Nilsson (46). The single-particle energies are given in units of $\hbar\bar{\omega}$, which represents the separation between major shells and, for ^{235}U, has the approximate value 6.6 MeV. The deformation parameter δ is a measure of the nuclear eccentricity; the value determined for ^{235}U, from the observed E2 transition moments, is $\delta \approx 0.25$. The single-particle states are labelled by the "asymptotic" quantum numbers $[\mathcal{N} n_3 \Lambda \Omega]$. The last quantum number Ω, which represents the component j_3, of the total angular momentum along the symmetry axis, is a constant of the motion for all values of 6. The additional quantum numbers refer to the structure of the orbits in the limit of large deformations, where they represent the total number of nodal surfaces (N), the number of nodal surfaces perpendicular to the symmetry axis (n_3), and the component of orbital angular momentum along the symmetry axis (Λ). Each orbit is doubly degenerate ($j_3 = \pm \Omega$), and a pairwise filling of orbits contributes no net angular momentum along the symmetry axis. For ^{235}U, with neutron number 143, it is seen that the lowest two configurations are expected to involve an odd neutron occupying the orbits [743 7/2] or [631 1/2], in agreement with the observed spectrum (see Fig. 6). It is also seen that the other observed low-lying bands in ^{235}U correspond to neighbouring orbits in the present figure.

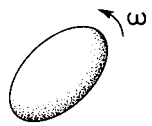
New possibilities for studying nuclear rotational motion were opened by the discovery of marked anisotropies in the angular distribution of fission fragments (52), which could be interpreted in terms of the rotational quantum numbers labelling the individual channels through which the fissioning nucleus passes the saddle-point shape (53). Present developments in the experimental tools hold promise of providing detailed information about band structure in the fission channels and thereby on rotational motion under circumstances radically different from those studied previously.

CONNECTION BETWEEN ROTATIONAL AND SINGLE-PARTICLE MOTION

The detailed testing of the rotational coupling scheme and the successful classification of intrinsic spectra provided a firm starting point for the next step in the development, which concerned the dynamics underlying the rotational motion.

The basis for this development was the bold idea of Inglis (54) to derive the moment of inertia by simply summing the inertial effect of each particle as it is dragged around by a uniformly rotating potential (see Fig. 8). In this approach, the potential appears to be externally "cranked", and the problems concerning the self-consistent origin for the rotating potential and the limitations of such a semi-classical description have continued over the years to be hotly debated issues. The discussion has clarified many points concerning the connection between collective and single-particle motion, but the basic idea of the cranking model has stood its tests to a remarkable extent (55, 35).

The evaluation of the moments of intertia on the basis of the cranking model gave the unexpected result that, for independent-particle motion, the moment would have a value approximately corresponding to rigid rotation (56). The fact that the observed moments were appreciably smaller than the rigid-body values could be qualitatively understood from the effect of the residual interactions that tend to bind the particles into pairs with angular momentum zero. A few years later, a basis for a systematic treatment of the moment of inertia with the inclusion of the many-body correlations associated with the pairing effect was given by Migdal (57) and Belyaev (58),



$$H = H_0 - \hbar\omega J_x$$

$$\mathcal{J} = 2\hbar^2 \sum_i \frac{\langle i|J_x|0\rangle^2}{E_i - E_0}$$

cranking model

Fig. 8. Nuclear moment of inertia from cranking model. The Hamiltonian ***H*** describing particle motion in a potential rotating with frequency ω about the x axis is obtained from the Hamiltonian H_o for motion in a fixed potential by the addition of the term proportional to the. component J_x of the total angular momentum, which represents the Coriolis and centrifugal forces acting in the rotating co-ordinate frame. The moment of inertia is obtained from a second-order perturbation treatment of this term and involves a sum over the excited states i. For independent-particle motion, the moment of inertia can be expressed as a sum of the contributions from the individual particles.

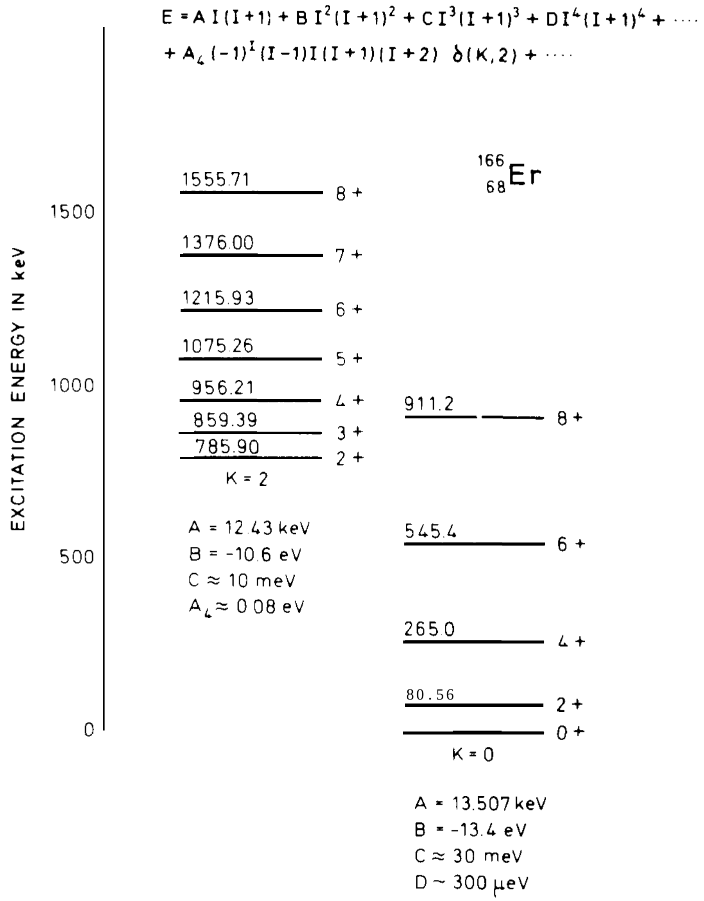


Fig. 9. Rotational bands in ^{166}Er. The figure is from (35) and is based on the experimental data by Reich and Cline (75). The bands are labelled by the component K of the total angular momentum with respect to the symmetry axis. The K = 2 band appears to represent the excitation of a mode of quadrupole vibrations involving deviations from axial symmetry in the nuclear shape.

electromagnetic transitions, β decay, particle transfer, etc. Thus, extensive measurements have been made of the E2 transitions between the two bands in ^{166}Er, and Fig. 10 shows the analysis of the empirical transition matrix elements in terms of the expansion in the angular momentum quantum numbers of initial and final states.

Such an analysis of the experimental data provides a phenomenological description of the rotational spectra in terms of a set of physically significant parameters. These parameters characterize the internal structure of the system with inclusion of the renormalization effects arising from the coupling to the rotational motion.

A systematic analysis of these parameters may be based on the ideas of the cranking model, and this approach has yielded important qualitative insight into the variety of effects associated with the rotational motion. However, in

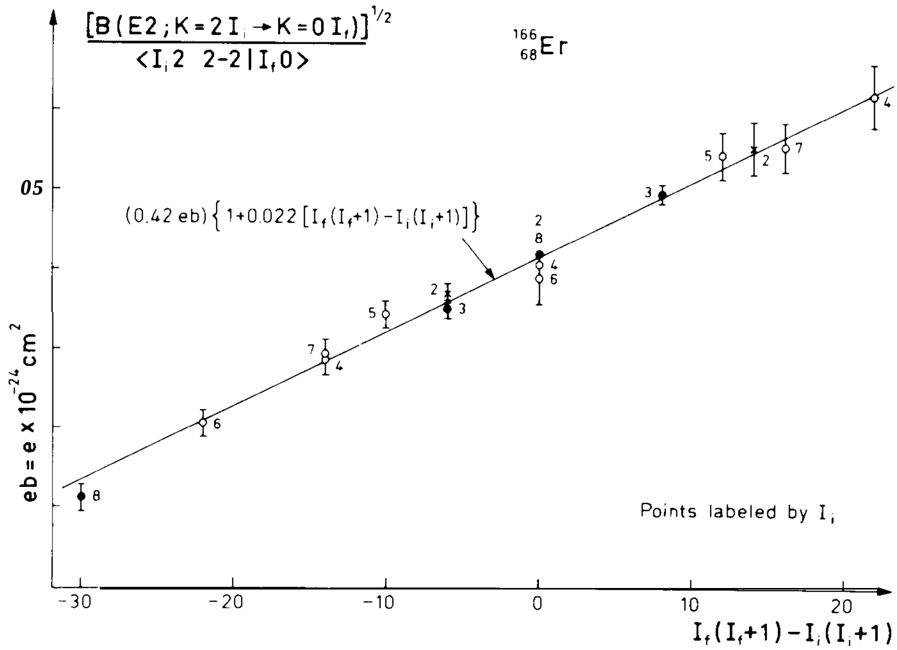


Fig. 10. Intensity relation for E2 transitions between rotational bands. The figure, which is from (35) and is based upon experimental data in (76), shows the measured reduced electric quadrupole transition probabilities B(E2) for transitions between members of the K = 2 and K = 0 bands in ^{166}Er (see Fig. 9). An expansion similar to that of the energies in Fig. 9, but taking into account the tensor properties of the E2 operator, leads to an expression for $(B(E2))^{1/2}$ which involves a Clebsch-Gordan coefficient $< I_i K_i 2 -2 | I_f K_f >$ (geometrical factor) multiplied by a power series in the angular momenta of I_i and I_f, of the initial and final states. The leading term in this expansion is a constant, and the next term is linear in $I_f(I_f+1)-I_i(I_i+1)$; the experimental data are seen to be rather well represented by these two terms.

this program, one faces significant unsolved problems. The basic coupling involved in the cranking model can be studied directly in the Coriolis coupling between rotational bands in ***odd-A*** nuclei associated with different orbits of the unpaired particle (77). The experiments have revealed, somewhat shockingly that, in many cases, this coupling is considerably smaller than the one directly experienced by the particles as a result of the nuclear rotation with respect to the distant galaxies (78). It is possible that this result may reflect an effect of the rotation on the nuclear potential itself (57, 79, 80, 35), but the problem stands as an open issue.

CURRENT PERSPECTIVES

In the years ahead, the study of nuclear rotation holds promising new perspectives. Not only are we faced with the problem already mentioned of a more deep-going probing of the rotational motion, which has become possible with the powerful modern tools of nuclear spectroscopy, but new frontiers are opening up through the possibility of studying nuclear states with very large

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