LOUIS DE BROGLIE

The wave nature of the electron

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When in 1920 I resumed my studies of theoretical physics which had long been interrupted by circumstances beyond my control, I was far from the idea that my studies would bring me several years later to receive such a high and envied prize as that awarded by the Swedish Academy of Sciences each year to a scientist: the Nobel Prize for Physics. What at that time drew me towards theoretical physics was not the hope that such a high distinction would ever crown my work; I was attracted to theoretical physics by the mystery enshrouding the structure of matter and the structure of radiations, a mystery which deepened as the strange quantum concept introduced by Planck in 1900 in his research on black-body radiation continued to encroach on the whole domain of physics.

To assist you to understand how my studies developed, I must first depict for you the crisis which physics had then been passing through for some twenty years.

For a long time physicists had been wondering whether light was composed of small, rapidly moving corpuscles. This idea was put forward by the philosophers of antiquity and upheld by Newton in the 18th century. After Thomas Young's discovery of interference phenomena and following the admirable work of Augustin Fresnel, the hypothesis of a granular structure of light was entirely abandoned and the wave theory unanimously adopted. Thus the physicists of last century spurned absolutely the idea of an atomic structure of light. Although rejected by optics, the atomic theories began making great headway not only in chemistry, where they provided a simple interpretation of the laws of definite proportions, but also in the physics of matter where they made possible an interpretation of a large number of properties of solids, liquids, and gases. In particular they were instrumental in the elaboration of that admirable kinetic theory of gases which, generalized under the name of statistical mechanics, enables a clear meaning to be given to the abstract concepts of thermodynamics. Experiment also yielded decisive proof in favour of an atomic constitution of electricity; the concept of the electricity corpuscle owes its appearance to Sir J. J. Thomson and you will all be familiar with H. A. Lorentz's use of it in his theory of electrons.

Some thirty years ago, physics was hence divided into two: firstly the physics of matter based on the concept of corpuscles and atoms which were supposed to obey Newton's classical laws of mechanics, and secondly radiation physics based on the concept of wave propagation in a hypothetical continuous medium, i.e. the light ether or electromagnetic ether. But these two physics could not remain alien one to the other; they had to be fused together by devising a theory to explain the energy exchanges between matter and radiation - and that is where the difficulties arose. While seeking to link these two physics together, imprecise and even inadmissible conclusions were in fact arrived at in respect of the energy equilibrium between matter and radiation in a thermally insulated medium: matter, it came to be said, must yield all its energy to the radiation and so tend of its own accord to absolute zero temperature! This absurd conclusion had at all costs to be avoided. By an intuition of his genius Planck realized the way of avoiding it: instead of assuming, in common with the classical wave theory, that a light source emits its radiation continuously, it had to be assumed on the contrary that it emits equal and finite quantities, *quanta*. The energy of each quantum has, moreover, a value proportional to the frequency v of the radiation. It is equal to hv, h being a universal constant since referred to as Planck's constant.

The success of Planck's ideas entailed serious consequences. If light is emitted as quanta, ought it not, once emitted, to have a granular structure? The existence of radiation quanta thus implies the corpuscular concept of light. On the other hand, as shown by Jeans and H. Poincaré, it is demonstrable that if the motion of the material particles in light sources obeyed the laws of classical mechanics it would be impossible to derive the exact law of black-body radiation, Planck's law. It must therefore be assumed that traditional dynamics, even as modified by Einstein's theory of relativity, is incapable of accounting for motion on a very small scale.

The existence of a granular structure of light and of other radiations was confirmed by the discovery of the photoelectric effect. If a beam of light or of X-rays falls on a piece of matter, the latter will emit rapidly moving electrons. The kinetic energy of these electrons increases linearly with the frequency of the incident radiation and is independent of its intensity. This phenomenon can be explained simply by assuming that the radiation is composed of quanta hv capable of yielding all their energy to an electron of the irradiated body: one is thus led to the theory of light quanta proposed by Einstein in 1905 and which is, after all, a reversion to Newton's corpuscular theory, completed by the relation for the proportionality between the energy of the corpuscles and the frequency. A number of arguments were put forward by Einstein in support of his viewpoint and in 1922 the discovery by A. H. Compton of the X-ray scattering phenomenon which bears his name confirmed it. Nevertheless, it was still necessary to adopt the wave theory to account for interference and diffraction phenomena and no way whatsoever of reconciling the wave theory with the existence of light corpuscles could be visualized.

As stated, Planck's investigations cast doubts on the validity of very small scale mechanics. Let us consider a material point which describes a small trajectory which is closed or else turning back on itself. According to classical dynamics there are numberless motions of this type which are possible complying with the initial conditions, and the possible values for the energy of the moving body form a continuous sequence. On the other hand Planck was led to assume that only certain preferred motions, *quantized* motions, are possible or at least stable, since energy can only assume values forming a discontinuous sequence. This concept seemed rather strange at first but its value had to be recognized because it was this concept which brought Planck to the correct law of black-body radiation and because it then proved its fruitfulness in many other fields. Lastly, it was on the concept of atomic motion quantization that Bohr based his famous theory of the atom; it is so familiar to scientists that I shall not summarize it here.

The necessity of assuming for light two contradictory theories-that of waves and that of corpuscles - and the inability to understand why, among the infinity of motions which an electron ought to be able to have in the atom according to classical concepts, only certain ones were possible: such were the enigmas confronting physicists at the time I resumed my studies of theoretical physics.

When I started to ponder these difficulties two things struck me in the main. Firstly the light-quantum theory cannot be regarded as satisfactory since it defines the energy of a light corpuscle by the relation $W = h\nu$ which contains a frequency ν. Now a purely corpuscular theory does not contain any element permitting the definition of a frequency. This reason alone renders it necessary in the case of light to introduce simultaneously the corpuscle concept and the concept of periodicity.

On the other hand the determination of the stable motions of the electrons in the atom involves whole numbers, and so far the only phenomena in which whole numbers were involved in physics were those of interference and of eigenvibrations. That suggested the idea to me that electrons themselves could not be represented as simple corpuscles either, but that a periodicity had also to be assigned to them too.

I thus arrived at the following overall concept which guided my studies: for both matter and radiations, light in particular, it is necessary to introduce the corpuscle concept and the wave concept at the same time. In other words the existence of corpuscles accompanied by waves has to be assumed in all cases. However, since corpuscles and waves cannot be independent because, according to Bohr's expression, they constitute two complementary forces of reality, it must be possible to establish a certain parallelism between the motion of a corpuscle and the propagation of the associated wave. The first objective to achieve had, therefore, to be to establish this correspondence.

With that in view I started by considering the simplest case: that of an isolated corpuscle, i.e. a corpuscle free from all outside influence. We wish to associate a wave with it. Let us consider first of all a reference system $Ox_0y_0z_0$ in which the corpuscle is immobile: this is the "intrinsic" system of the corpuscle in the sense of the relativity theory. In this system the wave will be stationary since the corpuscle is immobile: its phase will be the same at every point; it will be represented by an expression of the form sin $2\pi\nu_0(t_0-\tau_0)$; t_0 being the intrinsic time of the corpuscle and τ_0 a constant.

In accordance with the principle of inertia in every Galilean system, the corpuscle will have a rectilinear and uniform motion. Let us consider such a Galilean system and let $v = \beta c$ be the velocity of the corpuscle in this system; we shall not restrict generality by taking the direction of the motion as the x-axis. In compliance with Lorentz' transformation, the time t used by an observer of this new system will be associated with the intrinsic time t_0 by the relation:

$$t_0 = \frac{t - \frac{\beta x}{c}}{\sqrt{1-\beta^2}}$$

and hence for this observer the phase of the wave will be given by

$$\sin 2\pi \frac{\nu_0}{\sqrt{1-\beta^2}}\left(t - \frac{\beta x}{c} - \tau_0\right).$$

For him the wave will thus have a frequency:

$$\nu = \frac{\nu_0}{\sqrt{1 - \beta^2}}$$

and will propagate in the direction of the x-axis at the phase velocity:

$$V = \frac{c}{\beta} = \frac{c^2}{v}$$

By the elimination of b between the two preceding formulae the following relation can readily be derived which defines the refractive index of the vacuum *n* for the waves considered:

$$n = \sqrt{1 - \frac{\nu_0^2}{\nu^2}}$$

A <<group velocity>> corresponds to this <<law of dispersion>>. You will be aware that the group velocity is the velocity of the resultant amplitude of a group of waves of very close frequencies. Lord Rayleigh showed that this velocity *U* satisfies equation :

$$\frac{1}{U} = \frac{\partial(n\nu)}{\partial\nu}$$

Here $U = v$, that is to say that the group velocity of the waves in the system *xyxt* is equal to the velocity of the corpuscle in this system. This relation is of very great importance for the development of the theory.

The corpuscle is thus defined in the system *xyzt* by the frequency v and the phase velocity *V* of its associated wave. To establish the parallelism of which we have spoken, we must seek to link these parameters to the mechanical parameters, energy and quantity of motion. Since the proportionality between energy and frequency is one of the most characteristic relations of the quantum theory, and since, moreover, the frequency and the energy transform in the same way when the Galilean reference system is changed, we may simply write

$$\text{energy} = h \times \text{frequency}, \quad \text{or } W = h\nu$$

where h is Planck's constant. This relation must apply in all Galilean systems and in the intrinsic system of the corpuscle where the energy of the corpuscle, according to Einstein, reduces to its internal energy $m_o c^2$ (m_0 being the rest mass) we have

$$h\nu_o = m_o c^2$$

This relation defines the frequency ν_o as a function of the rest mass m_o, or inversely.

The quantity of movement is a vector p equal to

$$\frac{m_0 v}{\sqrt{1-\beta^2}}$$

and we have:

$$(p) = \frac{m_0 v}{\sqrt{1-\beta^2}} = \frac{Wv}{c^2} = \frac{h\nu}{V} = \frac{h}{\lambda}$$

The quantity; λ is the distance between two consecutive peaks of the wave, i.e. the "wavelength". Hence:

$$\lambda = \frac{h}{p}$$

This is a fundamental relation of the theory.

The whole of the foregoing relates to the very simple case where there is no field of force at all acting on the corpuscles. I shall show you very briefly how to generalize the theory in the case of a corpuscle moving in a constant field of force deriving from a potential function *F(xyz)*. By reasoning which I shall pass over, we are then led to assume that the propagation of the wave corresponds to a refractive index which varies from point to point in space in accordance with the formula:

$$n(xyz) = \sqrt{\left[1 - \frac{F(xyz)}{h\nu}\right]^2 - \frac{\nu_o^2}{\nu^2}}$$

or to a first approximation if the corrections introduced by the theory of relativity are negligible

$$n(xyz) = \sqrt{\frac{2(E-F)}{m_0c^2}}$$

with $E = W - m_0c^2$. The constant energy W of the corpuscle is still associated with the constant frequency ν of the wave by the relation

$$W = h\nu$$

while the wavelength λ which varies from one point to another of the force field is associated with the equally variable quantity of motion p by the following relation

$$\lambda(xyz) = \frac{h}{p(xyz)}$$

Here again it is demonstrated that the group velocity of the waves is equal to the velocity of the corpuscle. The parallelism thus established between the corpuscle and its wave enables us to identify Fermat's principle for the waves and the principle of least action for the corpuscles (constant fields). Fermat's principle states that the ray in the optical sense which passes through two points A and B in a medium having an index $n(xyz)$ varying from one point to another but constant in time is such that the integral $\int_A^B n\,dl$ taken along this ray is extreme. On the other hand Maupertuis' principle of least action teaches us the following: the trajectory of a corpuscle passing through two points A and B in space is such that the integral $\int_A^B p\,dl$ taken along the trajectory is extreme, provided, of course, that only the motions corresponding to a given energy value are considered. From the relations derived above between the mechanical and the wave parameters, we have:

$$n = \frac{c}{V} = \frac{c}{\nu}\cdot\frac{1}{\lambda} = \frac{c}{h\nu}\cdot\frac{h}{\lambda} = \frac{c}{W}p = \text{const.}\, p$$

since W is constant in a constant field. It follows that Fermat's and Maupertuis' principles are each a translation of the other and the possible trajectories of the corpuscle are identical to the possible rays of its wave.

These concepts lead to an interpretation of the conditions of stability introduced by the quantum theory. Actually, if we consider a closed trajectory C in a constant field, it is very natural to assume that the phase of the associated wave must be a uniform function along this trajectory. Hence we may write :

$$\int_C \frac{\mathrm{d}l}{\lambda} = \int_C \frac{1}{h} p\,\mathrm{d}l = \text{integer}$$

This is precisely Planck's condition of stability for periodic atomic motions. The conditions of quantum stability thus emerge as analogous to resonance phenomena and the appearance of integers becomes as natural here as in the theory of vibrating cords and plates.

The general formulae which establish the parallelism between waves and corpuscles may be applied to corpuscles of light on the assumption that here the rest mass m_0 is infinitely small. Actually, if for a given value of the energy W, m_0 is made to tend towards zero, v and V are both found to tend towards c and at the limit the two fundamental formulae are obtained on which Einstein had based his light-quantum theory

$$W = h\nu \qquad p = \frac{h\nu}{c}$$

Such are the main ideas which I developed in my initial studies. They showed clearly that it was possible to establish a correspondence between waves and corpuscles such that the laws of mechanics correspond to the laws of geometrical optics. In the wave theory, however, as you will know, geometrical optics is only an approximation: this approximation has its limits of validity and particularly when interference and diffraction phenomena are involved, it is quite inadequate. This prompted the thought that classical mechanics is also only an approximation relative to a vaster wave mechanics. I stated as much almost at the outset of my studies, i.e. "A new mechanics must be developed which is to classical mechanics what wave optics is to geometrical optics". This new mechanics has since been developed, thanks mainly to the fine work done by Schrödinger. It is based on wave propagation equations and strictly defines the evolution in time of the wave associated with a corpuscle. It has in particular succeeded in giving a new and more satisfactory form to the quantization conditions of intra-atomic motion since the classical quantization conditions are justified, as we have seen, by the application of geometrical optics to the waves associated with the intra-atomic corpuscles, and this application is not strictly justified.

I cannot attempt even briefly to sum up here the development of the new mechanics. I merely wish to say that on examination it proved to be identical with a mechanics independently developed, first by Heisenberg, then by Born, Jordan, Pauli, Dirac, etc.: quantum mechanics. The two mechanics, wave and quantum, are equivalent from the mathematical point of view.

We shall content ourselves here by considering the general significance of the results obtained. To sum up the meaning of wave mechanics it can be stated that: "A wave must be associated with each corpuscle and only the study of the wave's propagation will yield information to us on the successive positions of the corpuscle in space". In conventional large-scale mechanical phenomena the anticipated positions lie along a curve which is the trajectory in the conventional meaning of the word. But what happens if the wave does not propagate according to the laws of optical geometry, if, say, there are interferences and diffraction? Then it is no longer possible to assign to the corpuscle a motion complying with classical dynamics, that much is certain. Is it even still possible to assume that at each moment the corpuscle occupies a well-defined position in the wave and that the wave in its propagation carries the corpuscle along in the same way as a wave would carry along a cork? These are difficult questions and to discuss them would take us too far and even to the confines of philosophy. All that I shall say about them here is that nowadays the tendency in general is to assume that it is not constantly possible to assign to the corpuscle a well-defined position in the wave. I must restrict myself to the assertion that when an observation is carried out enabling the localization of the corpuscle, the observer is invariably induced to assign to the corpuscle a position in the interior of the wave and the probability of it being at a particular point M of the wave is proportional to the square of the amplitude, that is to say the intensity at M.

This may be expressed in the following manner. If we consider a cloud of corpuscles associated with the same wave, the intensity of the wave at each point is proportional to the cloud density at that point (i.e. to the number of corpuscles per unit volume around that point). This hypothesis is necessary to explain how, in the case of light interferences, the light energy is concentrated at the points where the wave intensity is maximum: if in fact it is assumed that the light energy is carried by light corpuscles, photons, then the photon density in the wave must be proportional to the intensity.

This rule in itselfwill enable us to understand how it was possible to verify the wave theory of the electron by experiment.

Let us in fact imagine an indefinite cloud of electrons all moving at the same velocity in the same direction. In conformity with the fundamental ideas of wave mechanics we must associate with this cloud an indefinite plane wave of the form

$$a \sin 2\pi \left[\frac{W}{h} t - \frac{\alpha x + \beta y + \gamma z}{\lambda}\right]$$

where $\alpha\beta\gamma$ are the cosines governing the propagation direction and where the wavelength λ is equal to h/p. With electrons which are not extremely fast, we may write

$$p = m_0 v$$

and hence

$$\lambda = \frac{h}{m_0 v}$$

where m_0 *is the* rest mass of the electron.

You will be aware that in practice, to obtain electrons moving at the same velocity, they are made to undergo a drop in potential P and we have

$$\frac{1}{2} m_0 v^2 = eP$$

Hence,

$$\lambda = \frac{h}{\sqrt{2 m_0 e P}}$$

Numerically this gives

$$\lambda = \frac{12.24}{\sqrt{P}} 10^{-8} \text{ cm} \quad (P \text{ in volts})$$

Since it is scarcely possible to use electrons other than such that have undergone a voltage drop of at least some tens of volts, you will see that the wavelength λ predicted by theory is at most of the order of 10^{-8} cm, i.e. of the order of the Ångström unit. It is also the order of magnitude of X-ray wavelengths.

Since the wavelength of the electron waves is of the order of that of X-rays, it must be expected that crystals can cause diffraction of these waves completely analogous to the Laue phenomenon. Allow me to refresh your memories what is the Laue phenomenon. A natural crystal such as rock salt, for example, contains nodes composed of the atoms of the substances making up the crystal and which are regularly spaced at distances of the order of an Ångström. These nodes act as diffusion centres for the waves and if the crystal is impinged upon by a wave, the wavelength of which is also of the order of an Ångström, the waves diffracted by the various nodes are in phase agreement in certain well-defined directions and in these directions the total diffracted intensity is a pronounced maximum. The arrangement of these diffraction maxima is given by the nowadays well-known mathematical theory developed by von Laue and Bragg which defines the position of the maxima as a function of the spacing of the nodes in the crystal and of the wavelength of the incident wave. For X-rays this theory has been admirably confirmed by von Laue, Friedrich, and Knipping and thereafter the diffraction of X-rays in crystals has become a commonplace experience. The accurate measurement of X-ray wavelengths is based on this diffraction: is there any need to remind this in the country where Siegbahn and co-workers are continuing their fine work?

For X-rays the phenomenon of diffraction by crystals was a natural consequence of the idea that X-rays are waves analogous to light and differ from it only by having a smaller wavelength. For electrons nothing similar could be foreseen as long as the electron was regarded as a simple small corpuscle. However, if the electron is assumed to be associated with a wave and the density of an electron cloud is measured by the intensity of the associated wave, then a phenomenon analogous to the Laue phenomenon ought to be expected for electrons. The electron wave will actually be diffracted intensely in the directions which can be calculated by means of the Laue-Bragg theory from the wavelength $\lambda = h/mv$, which corresponds to the known velocity v of the electrons impinging on the crystal. Since, according to our general principle, the intensity of the diffracted wave is a measure of the density of the cloud of diffracted electrons, we must expect to find a great many diffracted electrons in the directions of the maxima. If the phenomenon actually exists it should thus provide decisive experimental proof in favour of the existence of a wave associated with the electron with wavelength *h/mv*, and so the fundamental idea of wave mechanics will rest on firm experimental foundations.

Now, experiment which is the final judge of theories, has shown that the phenomenon of electron diffraction by crystals actually exists and that it obeys exactly and quantitatively the laws of wave mechanics. To Davisson and Germer, working at the Bell Laboratories in New York, falls the honour of being the first to observe the phenomenon by a method analogous to that of von Laue for X-rays. By duplicating the same experiments but replacing the single crystal by a crystalline powder in conformity with the method introduced for X-rays by Debye and Scherrer, Professor G. P. Thomson of Aberdeen, son of the famous Cambridge physicist Sir J. J. Thomson, found the same phenomena. Then Rupp in Germany, Kikuchi in Japan, Ponte in France and others reproduced them, varying the experimental conditions. Today, the existence of the phenomenon is beyond doubt and the slight difficulties of interpretation posed by the first experiments of Davisson and Germer appear to have been satisfactorily solved.

Rupp has even managed to bring about electron diffraction in a particularly striking form. You will be familiar with what are termed diffraction gratings in optics: these are glass or metal surfaces, plane or slightly curved, on which have been mechanically traced equidistant lines, the spacing between which is comparable in order of magnitude with the wavelengths of light waves. The waves diffracted by these lines interfere, and the interferences give rise to maxima of diffracted light in certain directions depending on the interline spacing, on the direction of the light impinging on the grating, and on the wavelength of this light. For a long time it proved impossible to achieve similar phenomena with this type of man-made diffraction grating using X-rays instead of light. The reason was that the wavelength of X-rays is much smaller than that of light and no instrument can draw lines on a surface, the spacing between which is of the order of magnitude of X-ray wavelengths. A number of ingenious physicists (Compton, J. Thibaud) found how to overcome the difficulty. Let us take an ordinary optical diffraction grating and observe it almost tangentially to its surface. The lines of the grating will appear to us much closer together than they actually are. For X-rays impinging at this almost skimming incidence on the grating the effect will be as if the lines were very closely set and diffraction phenomena analogous to those of light will occur. This is what the above-mentioned physicists confirmed. But then, since the electron wavelengths are of the order of X-ray wavelengths, it must also be possible to obtain diffraction phenomena by directing a beam of electrons on to an optical diffraction grating at a very low angle. Rupp succeeded in doing so and was thus able to measure the wavelength of electron waves by comparing them directly with the spacing of the mechanically traced lines on the grating.

Thus to describe the properties of matter as well as those of light, waves and corpuscles have to be referred to at one and the same time. The electron can no longer be conceived as a single, small granule of electricity; it must be associated with a wave and this wave is no myth; its wavelength can be measured and its interferences predicted. It has thus been possible to predict a whole group of phenomena without their actually having been discovered. And it is on this concept of the duality of waves and corpuscles in Nature, expressed in a more or less abstract form, that the whole recent development of theoretical physics has been founded and that all future development of this science will apparently have to be founded.