Uncertainty Outside and Inside Economic Models
Nobel Lecture

Lars Peter Hansen
University of Chicago

December 8, 2013
Skepticism

Le doute n’est pas une condition agréable, mais la certitude est absurde. Voltaire (1776)
Components of Uncertainty

- Risk - probabilities assigned by a given model

- Ambiguity - not knowing which among a family of models should be used to assess risk

Skepticism about the model specification
Researcher and Investor Uncertainty

- **Researchers outside a model**
  Given a dynamic economic model:
  - estimate unknown parameters;
  - assess model implications.

- **Investors inside a model**
  In constructing a dynamic economic model:
  - depict economic agents (consumers, enterprises, policy makers) as they cope with uncertainty;
  - construct equilibrium interactions that acknowledge this uncertainty.
Overview: Techniques and Applications

- Time series econometrics and rational expectations
- Generalized Method of Moments estimation, applications and extensions
- Empirical challenges
- Uncertainty and investors inside the model
- Uncertainty and policy
Time Series Econometrics and Rational Expectations

- Bachelier (1901) - Slutsky (1926) - Yule (1927): random shocks are impulses for time series.
  - finance
  - macroeconomics

- Frisch (1933) - Haavelmo (1943): dynamic models provide a formal connection between economic inputs and statistical methods used outside the model.

- Muth (1961) - Lucas (1972): economic agents inside the model have rational expectations.
Rational Expectations Econometrics

- Expectations determined *inside* the model.
- A new form of econometric restrictions.
- **Challenge**: Requires a complete model specification including a specification of the information available to the economic agents *inside* the model.

Early work by Sargent (1973) and others, and my initial publication Hansen and Sargent (1980).
Doing Something without Doing Everything

- Generalized Method of Moments estimation

- Study partially specified models that link financial markets and the macroeconomy.

- Build and extend an earlier econometrics literature on estimating equations in a simultaneous system, in particular Sargan (1958, 1959).
Model the investment in risky capital and the pricing of financial assets:

$$E \left[ \left( \frac{S_{t+\ell}}{S_t} \right) X_{t+\ell} \left| \mathcal{F}_t \right. \right] = Q_t$$

where

- $S$ is a stochastic discount factor (SDF) process;
- $X_{t+\ell}$ vector of payoffs on physical or financial assets;
- $\ell$ is the investment horizon;
- $Q_t$ vector of asset prices;
- $\mathcal{F}_t$ is the investor information;
- $E$ is the expectation implied by the data generating process and used by investors inside the model.
Doing Something without Doing Everything

- Recall

\[ E \left[ \left( \frac{S_{t+\ell}}{S_t} \right) X_{t+\ell} - Q_t \bigg| \mathcal{F}_t \right] = 0. \]

- \( Z_t \): variables in the investor information set \( \mathcal{F}_t \). Then

\[ E \left[ \left( \frac{S_{t+\ell}}{S_t} \right) X_{t+\ell} Z_t - Q_t Z_t \right] = 0. \]

Observations:

- SDF depends on data and model parameters;
- Approximate expectations by time series averages;
- Build and justify formal methods for estimation and inference;
- Avoid a complete specification of investor information;
- Extend to other applications: estimate and assess misspecified models.
Further Econometric Challenges

- Formal study of an entire class of estimators:
  - pose as a semi-parametric estimation problem;
  - construct a well defined efficiency bound for the class of the many possible estimators.

  Hansen (1985) and Chamberlain (1987)

- Related approaches:
  - Ignore parametric representation of the SDF. Empirical pricing restrictions are consistent with many SDF’s. Hansen and Jaganathan (1991), Luttmer (1996)
Applications to Empirical Finance

Hansen and co-authors

- Hodrick (1980, 1983) - characterizing risk premia in forward foreign exchange market;
- Singleton (1982, 1983) - macro finance linkages implied by the SDF for macroeconomists’ “typical” model of investors;
- Richard (1987) - conditioning information and risk-return tradeoffs given a “general specification” of SDFs;
The Changing Price of Uncertainty

Stochastic discount factors encode compensations for exposure to risk: risk prices.

Finding: “risk price” channel provides a predictable and important source for variation observed in security markets.

- SDF’s are highly variable.
- Volatility is conditional on information pertinent to investors.
- Volatility is higher in bad macroeconomic times than good ones. Campbell-Cochrane (1999).

*Modeling challenge*: What is the source of this SDF volatility?

*Possible explanation*: Investor concern about misspecification inside a dynamic economic model.
Asset Pricing under a Belief Distortion

\[
\tilde{E} \left[ \left( \frac{\tilde{S}_{t+\ell}}{\tilde{S}_t} \right) X_{t+\ell} \mid \mathcal{F}_t \right] = Q_t
\]

(1)

where \( \tilde{E} \) is the distorted expectation operator and \( \tilde{S} \) is the corresponding stochastic discount factor.

- Convenient to represent distorted beliefs using a positive martingale \( M \) with a unit expectation via the formula:

\[
\tilde{E} [Y_{t+\ell} \mid \mathcal{F}_t] = E \left[ \left( \frac{M_{t+\ell}}{M_t} \right) Y_{t+\ell} \mid \mathcal{F}_t \right].
\]

- Rewrite (1) as:

\[
E \left[ \left( \frac{M_{t+\ell} \tilde{S}_{t+\ell}}{M_t \tilde{S}_t} \right) X_{t+\ell} \mid \mathcal{F}_t \right] = E \left[ \left( \frac{S_{t+\ell}}{S_t} \right) X_{t+\ell} \mid \mathcal{F}_t \right] = Q_t
\]

where \( S = M \tilde{S} \).
Asset Pricing under a Belief Distortion

SDF representation

\[ S = M \tilde{S} \]

- \( \tilde{S} \) constructed from data and model parameters.
- \( M \) is a likelihood ratio.
- When \( M \) close to one, the distortion is small.
- Statistical criteria provide interpretable measures of the magnitude of the distortion.

When the distortion is small, a statistician with a large number of observations will struggle to tell the difference between two models.
Statistical Quantification as a Guide for Modeling

\[ S = M \tilde{S} \]

\[ \text{distorted risk beliefs preference} \]

Statistical tools support a refinement of rational expectations \((M = 1)\).

- **Inspiration:** detect when historical evidence is less informative;
- **Discipline:** limit the scope of belief distortions such as:
  - animal spirits
  - heterogeneous beliefs
  - subjective concerns about rare events
  - overconfidence
Modeling Challenges

\[ S = M \tilde{S} \]

- **distorted risk beliefs**
- **preference**

**Challenges:**

- Add structure and content to belief distortions.
- Make the belief distortions a formal source for fluctuating uncertainty prices.

**Approach:** model misspecification and uncertainty more broadly conceived.
Components of Uncertainty

- Risk: a distribution for next periods outcome $Y$ given this periods state $X$ indexed by a parameter $\theta$. Represent as a density $\phi( |x, \theta )$.
- Ambiguity: a family $\Pi$ of probability distributions $\pi$ over $\theta$.
- Reduction: a unique $\pi$ and average over models.

$$\bar{\phi}( |x) = \int \phi( |x, \theta) \pi(d\theta)$$

- Robustness: a family $\Pi$ and explore utility consequences of alternative $\pi$'s. Implemented by a distorted model average.
Operationalizing Robustness and Ambiguity Aversion

Conceptual apparatus:

- Explore a family of perturbations to a model subject to constraints or penalization. (Origins in control theory)
- Explore a family of “posteriors/priors” used to weight models. Dynamic and robust extension of Bayesian decision theory. (Origins in statistics)

What is available:

- Extensions of Savage’s axiomatic foundations.
- Tractable representations.
Enriching the Uncertainty Pricing Dynamics

- Two reasons for *skepticism* about models:
  - some future model variations *cannot* be inferred from past evidence;
  - while some features of models *can* be inferred from past evidence there remains prior ambiguity.

- Outcome: Uncertainty in the persistence of macroeconomic growth. *High persistence* is bad in *bad* times and *low persistence* is bad in *good* times. This becomes a source for *ex post* distortions in beliefs and uncertainty prices that change over time in interesting ways.

- Explicit model of $M$ and thus $S$ that depends on macroeconomic shocks, state vector and model parameters where:

$$ S = M \tilde{S} $$

- distorted risk
- beliefs preference
Uncertainty and Policy Implications

Two approaches

- Uncertainty outside structural econometric models;

- Equilibrium interactions within a model when policy makers and the private sector simultaneously confront uncertainty.
Implications for Financial Oversight

- **Systemic risk**: a grab bag of scenarios rationalizing interventions in financial markets.

- Haldane (Bank of England), Tarullo (Board of Governors): Limited understanding of systemic risk challenges its value as a guiding principle for financial oversight!

- **Systemic uncertainty**

- Complicated problems do not necessarily require complicated solutions.