Uncertainty Outside and Inside Economic Models

Nobel Lecture

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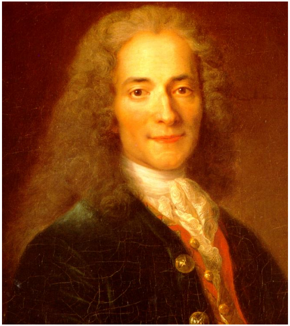
Skepticism



Le doute n'est pas une condition agréable, mais la certitude est absurde. Voltaire (1776)

Components of Uncertainty

- Risk - probabilities assigned by a given model
- Ambiguity - not knowing which among a family of models should be used to assess risk

Skepticism about the model specification

Researcher and Investor Uncertainty

- *Researchers outside a model*
  Given a dynamic economic model:
  - estimate unknown parameters;
  - assess model implications.

- *Investors inside a model*
  In constructing a dynamic economic model:
  - depict economic agents (consumers, enterprises, policy makers) as they cope with uncertainty;
  - construct equilibrium interactions that acknowledge this uncertainty.

Overview: Techniques and Applications

- Time series econometrics and rational expectations
- Generalized Method of Moments estimation, applications and extensions
- Empirical challenges
- Uncertainty and investors inside the model
- Uncertainty and policy

Time Series Econometrics and Rational Expectations

- Bachelier (1901) - Slutsky (1926) -Yule (1927): random shocks are impulses for time series.
  - finance
  - macroeconomics
- Frisch (1933) - Haavelmo (1943): dynamic models provide a formal connection between economic inputs and statistical methods used *outside* the model.
- Muth (1961) - Lucas (1972): economic agents *inside* the model have rational expectations.

Rational Expectations Econometrics

- Expectations determined *inside* the model.
- A new form of econometric restrictions.
- Challenge: Requires a complete model specification including a specification of the information available to the economic agents *inside* the model.

Early work by Sargent (1973) and others, and my initial publication Hansen and Sargent (1980).

Doing Something without Doing Everything

- Generalized Method of Moments estimation
- Study partially specified models that link financial markets and the macroeconomy.
- Build and extend an earlier econometrics literature on estimating equations in a simultaneous system, in particular Sargan (1958, 1959).

Doing Something without Doing Everything

Model the investment in risky capital and the pricing of financial assets:

$$E\left[\left(\frac{S_{t+\ell}}{S_t}\right) X_{t+\ell} \middle| \mathcal{F}_t\right] = Q_t$$

where

- S is a stochastic discount factor (SDF) process;
- $X_{t+\ell}$ vector of payoffs on physical or financial assets;
- ℓ is the investment horizon;
- Q_t vector of asset prices;
- $\mathcal{F}_t$ is the investor information;
- E is the expectation implied by the data generating process and used by investors *inside the model*.

Doing Something without Doing Everything

- Recall
$$E\left[\left(\frac{S_{t+\ell}}{S_t}\right)X_{t+\ell} - Q_t \,\middle|\, \mathcal{F}_t\right] = 0.$$
- Z_t: variables in the investor information set $\mathcal{F}_t$. Then
$$E\left[\left(\frac{S_{t+\ell}}{S_t}\right)X_{t+\ell}Z_t - Q_tZ_t\right] = 0.$$

Observations:

- SDF depends on data and model parameters;
- Approximate expectations by time series averages;
- Build and justify formal methods for estimation and inference;
- Avoid a complete specification of investor information;
- Extend to other applications: estimate and assess misspecified models.

Further Econometric Challenges

- Formal study of an entire class of estimators:
  - pose as a semi-parametric estimation problem;
  - construct a well defined efficiency bound for the class of the many possible estimators.

  Hansen (1985) and Chamberlain (1987)
- Related approaches:
  - Ignore parametric representation of the SDF. Empirical pricing restrictions are consistent with many SDF's. Hansen and Jaganathan (1991), Luttmer (1996)
  - SDF model misspecified. A different perspective on estimation and model comparison. Hansen and Jaganathan (1997), Hansen, Heaton and Luttmer (1995)

Applications to Empirical Finance

Hansen and co-authors

- Hodrick (1980,1983) - characterizing risk premia in forward foreign exchange market;
- Singleton (1982,1983) - macro finance linkages implied by the SDF for macroeconomists' "typical" model of investors;
- Richard (1987) - conditioning information and risk -return tradeoffs given a "general specification" of SDFs;
- Jagannathan (1991) and Cochrane (1992) - empirical characterizations of SDF's without parametric restrictions.

Hodrick

Singleton

Richard

Jagannathan

Cochrane

The Changing Price of Uncertainty

Stochastic discount factors encode compensations for exposure to risk: risk prices.

Finding: "risk price" channel provides a predictable and important source for variation observed in security markets.

- SDF's are highly variable.
- Volatility is conditional on information pertinent to investors.
- Volatility is higher in bad macroeconomic times than good ones. Campbell-Cochrane (1999).

Modeling challenge: What is the source of this SDF volatility?

Possible explanation: Investor concern about misspecification inside a dynamic economic model.

Asset Pricing under a Belief Distortion

$$\widetilde{E}\left[\left(\frac{\widetilde{S}_{t+\ell}}{\widetilde{S}_t}\right) X_{t+\ell}\middle|\, \mathcal{F}_t\right] = Q_t \tag{1}$$

where $\widetilde{E}$ is the distorted expectation operator and $\widetilde{S}$ is the corresponding stochastic discount factor.

- Convenient to represent distorted beliefs using a positive martingale M with a unit expectation via the formula:

$$\widetilde{E}\left[Y_{t+\ell}|\mathcal{F}_t\right] = E\left[\left(\frac{M_{t+\ell}}{M_t}\right) Y_{t+\ell}\middle|\, \mathcal{F}_t\right].$$

- Rewrite (1) as:

$$E\left[\left(\frac{M_{t+\ell}\widetilde{S}_{t+\ell}}{M_t\widetilde{S}_t}\right) X_{t+\ell}\middle|\, \mathcal{F}_t\right] = E\left[\left(\frac{S_{t+\ell}}{S_t}\right) X_{t+\ell}\middle|\, \mathcal{F}_t\right] = Q_t$$

where $S = M\widetilde{S}$.

Asset Pricing under a Belief Distortion

SDF representation

$$S = \underbrace{M}_{\text{distorted beliefs}} \quad \underbrace{\widetilde{S}}_{\text{risk preferences}}$$

- $\widetilde{S}$ constructed from data and model parameters.
- M is a likelihood ratio.
- When M close to one, the distortion is small.
- Statistical criteria provide interpretable measures of the magnitude of the distortion.

When the distortion is small, a statistician with a large number of observations will struggle to tell the difference between two models.

Statistical Quantification as a Guide for Modeling

$$S = \underset{\substack{\text{distorted} \\ \text{beliefs}}}{M} \quad \underset{\substack{\text{risk} \\ \text{preference}}}{\widetilde{S}}$$

Statistical tools support a refinement of rational expectations ($M = 1$).

- Inspiration: detect when historical evidence is less informative;
- Discipline: limit the scope of belief distortions such as:
  - animal spirits
  - heterogeneous beliefs
  - subjective concerns about rare events
  - overconfidence

Modeling Challenges

$$\underset{\substack{\text{distorted}\\\text{beliefs}}}{} \quad S = \underbrace{M}_{\substack{\text{distorted}\\\text{beliefs}}} \quad \underbrace{\widetilde{S}}_{\substack{\text{risk}\\\text{preference}}}$$

- Challenges:
  - Add structure and content to belief distortions.
  - Make the belief distortions a formal source for fluctuating uncertainty prices.

- Approach: model misspecification and uncertainty more broadly conceived.

Components of Uncertainty

- Risk: a distribution for next periods outcome Y given this periods state X indexed by a parameter θ. Represent as a density $\phi(\ |x, \theta)$.
- Ambiguity: a family Π of probability distributions π over θ.
- Reduction: a unique π and average over models.

$$\bar{\phi}(\ |x) = \int \phi(\ |x, \theta)\pi(d\theta)$$

- Robustness: a family Π and explore utility consequences of alternative π's. Implemented by a distorted model average.

Knight (1921)

Wald (1939)

de Finetti (1937)

Savage (1954)

Operationalizing Robustness and Ambiguity Aversion

Conceptual apparatus:

- Explore a family of perturbations to a model subject to constraints or penalization. (Origins in control theory)
- Explore a family of "posteriors/priors" used to weight models. Dynamic and robust extension of Bayesian decision theory. (Origins in statistics)

What is available:

- Extensions of Savage's axiomatic foundations.
- Tractable representations.

Enriching the Uncertainty Pricing Dynamics

- Two reasons for *skepticism* about models:
  - some future model variations cannot be inferred from past evidence;
  - while some features of models can be inferred from past evidence there remains prior ambiguity.
- Outcome: Uncertainty in the persistence of macroeconomic growth. High persistence is bad in bad times and low persistence is bad in good times. This becomes a source for *ex post* distortions in beliefs and uncertainty prices that change over time in interesting ways.
- Explicit model of M and thus S that depends on macroeconomic shocks, state vector and model parameters where:

$$S = \underset{\substack{\text{distorted}\\ \text{beliefs}}}{M} \quad \underset{\substack{\text{risk}\\ \text{preference}}}{\widetilde{S}}$$

Uncertainty and Policy Implications

Two approaches

- Uncertainty outside structural econometric models;
- Equilibrium interactions within a model when policy makers and the private sector simultaneously confront uncertainty.

Implications for Financial Oversight

- Systemic risk: a grab bag of scenarios rationalizing interventions in financial markets.
- Haldane (Bank of England), Tarullo (Board of Governors): Limited understanding of systemic risk challenges its value as a guiding principle for financial oversight!
- Systemic uncertainty
- Complicated problems do not necessarily require complicated solutions.