Magneto Oscillations in unconventional superconductors well below H_{c2}

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Abstract

We have studied the De Haas-van Alphen oscillations of unconventional superconductor for magnetic field well below H_{c2} . We find that the amplitude of the oscillation is increased substantially relative to that of an s-wave gap material.

PACS numbers: 74.20.Fg, 72.15.Gd, 74.60.-w

The De Haas-van Alphen Effect (dHvA) has been intensively studied since it was first discovered in Nb_2Sn at fields well below H_{c2} [1]. Later the effect was seen in V_3Si, Nb_3Ge and borocarbides where $\omega_c\tau \gg 1$, (see [2] for a review). Most theoretical efforts [3-9] have focused on $H_{c2} - H \ll H_{c2}$. Here the oscillations are considered to be due to a non zero normal density of states at the Fermi level which continued to exist below H_{c2} .

We study this effect in the field range in which the spacing d between vortices is large compared to the ξ size of the vortex cores. In addition d is small compared to the penetration depth of δ_L. In this regime, the magnetic field is essentially uniform and the gap Δ has nearly a constant value since the area covered by the vortex cores, $\pi\xi^2$, is small compared to the area d^2 between vortices.

The peak in the density of states of the s-wave superconductor at Δ leads to an exponential reduction of the dHvA amplitude [7]. If the gap has lines

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of zeros along the Fermi surface one might expect that an increase of the dHvA amplitude would occur [10]. For a d-wave superconductor the gap is zero at some points on the Fermi surface. In a magnetic Field, this leads to a finite density of states at the chemical potential [11].

We have studied the problem in two situations. For lines of zeros, we find that the inverse Dingle temperature is reduced from Δ to $\Delta\xi_0/d$ leading to an exponentially enhanced amplitude. For point zeros, a topologic property excludes level crossing at the chemical potential and therefore a small dHvA amplitude.

In the above field range the gap is given by

$$\Delta(\mathbf{r}) = \Delta e^{i\phi(\mathbf{r})} \tag{1}$$

The super currents outside the vortex cores are

$$\mathbf{j}(\mathbf{r}) = -\frac{c}{4\pi}\delta_L^{-2}\left(\mathbf{A} - \frac{c}{2e}\nabla\phi\right) \equiv -\frac{c}{4\pi}\delta_L^{-2}\mathbf{Q} \tag{2}$$

The following equations result from the topological singularity of the phase where the gap goes to zero.

$$\begin{aligned} \nabla \times \mathbf{Q} &= \mathbf{B} - \phi_0 \sum_i \delta(\mathbf{r} - \mathbf{r}_i), \\ \mathbf{B} - \delta_L^2 \Delta \mathbf{B} &= \phi_0 \sum_i \delta(\mathbf{r} - \mathbf{r}_i). \end{aligned} \tag{3}$$

For a square lattice $[\mathbf{r}_i = d(n,m)]$

$$B(\mathbf{r}) = \frac{\phi_0}{d^2}\sum_{\mathbf{k}} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2\delta_L^2 + 1}; \quad \left(\mathbf{k} = \frac{2\pi}{d}(l,k)\right). \tag{4}$$

It follows from (2) and (4) that

$$\bar{B} = \phi_0 d^{-2}, \quad |\mathbf{Q}| \sim \phi_0/d, \quad \Delta B \sim \bar{B}(d/\delta_L)^2 \tag{5}$$

where $\bar{B}$ is the average field and ΔB its spatial fluctuation. We consider a two dimensional Fermi surface.

The oscillatory part of the magnetization M is given by

$$M = -\frac{\mu}{B}\mathrm{Tr}\left[T\sum_n G_{11}(i\omega_n)\right] \tag{6}$$

Spur denotes the trace over all states, including spin of the quantity $G_{11}(z)$ of the matrix

$$\hat{G}(z) = \begin{pmatrix} G(z) & F(z) \\ F^{\dagger}(z) & -G(z) \end{pmatrix} \tag{7}$$

$\hat{G}$ satisfies the Gor'kov equation

$$\begin{pmatrix} z - (\hat{H} - \mu) & \Delta \\ -\Delta & -z - (\hat{\hat{H}} - \mu) \end{pmatrix} \hat{G}(z) = \hat{1} \tag{8}$$

Here, the spatial dependence of the phase $\phi(\mathbf{r})$ has been removed by a gauge transformation. Nevertheless, the gap $\Delta = \Delta(\mathbf{p})$ may depend on the direction of p along the Fermi surface.

For simplicity we consider a cylindrical Fermi surface. The Hamiltonians in (8) are given by

$$\hat{\hat{H}}, \hat{H} = \frac{1}{2m}\left(\mathbf{p} \pm \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2 \tag{9}$$

The magnetic field outside the cores and the part of $\mathbf{A}(\mathbf{r})$ [or $bfQ(\mathbf{r})$ responsible for the periodic variations are given by (3) and (4), with the average field given by $\bar{\mathbf{B}} = \boldsymbol{\nabla} \times \mathbf{A_0}$. Thus, we have

$$\mathbf{A}(\mathbf{r}) = \mathbf{A}_0(\mathbf{r}) + \mathbf{Q}(\mathbf{r}) \tag{10}$$

and employ the Landau gauge $\mathbf{A}_{x0} = -\bar{B}y$. Since Q is small we retain only terms first order in Q

$$\hat{\hat{H}}, \hat{H} = \frac{1}{2m}\left(\mathbf{p} \pm \frac{e}{c}\mathbf{A}_0(\mathbf{r})\right)^2 \pm \frac{e}{c}\mathbf{v}_F \cdot \mathbf{Q}(\mathbf{r}) \tag{11}$$

From (5) it follows that

$$|(ev_F/c)Q| \sim v_F/d \ll \Delta; \quad (d \gg \xi_0) \tag{12}$$

In the Landau gauge p_x is conserved, as in (8) since the gap couples p_x and $-p_x$. Thus we have

$$\hat{\hat{H}}, \hat{H} \simeq \frac{1}{2m}\left(p_x^2 + p_y^2\right) \pm \frac{e}{mc}\left(p_x Q_x + p_y Q_y\right) \tag{13}$$

where $\hat{p}_x$, $\hat{p}_y$, are the kinetic momentum operators $[\hat{p}_x, \hat{p}_y] = ie\bar{B}/c$. We will carry out calculations in the momentum representations so

$$A_{0x} = -i\bar{B}(d/dp_y) \tag{14}$$

The unperturbed Hamiltonian $Q = 0$ in (13) has eigenvalues $E_n = \omega_c(n + \frac{1}{2})$ with real eigenfunctions which we use in calculating the trace in (6). In (13), the terms linear in Q and the gap terms lead to nondiagonal terms between free electron eigenstates for the $\hat{G}$ matrix of the form

$$\sum_{N'} \Delta_{N,N'} F^{\dagger}_{N'\bar{N}}, \quad \sum_{N'} \Delta_{N,N'} G_{N'\bar{N}} \tag{15}$$

if $\Delta(\mathbf{p})$ is anisotropic.

For weak coupling $\Delta(\mathbf{p})$ depends only on the direction of $\mathbf{p}$ along the Fermi surface. For example, for d-wave pairing $\Delta \propto p_x^2 - p_y^2$ so that

$$\begin{aligned} \Delta(\mathbf{p}) &= \Delta_0 \cos 2\varphi & (16)\\ &= \Delta_0(1 - 2p_y^2/p_F^2) & (17) \end{aligned}$$

Since the only nonzero matrix elements of $\hat{p}_y$ are

$$(\hat{p}_y)_{N,N-1} = (\hat{p}_y)_{N-1,N} = \sqrt{Ne\bar{B}/2c}. \tag{18}$$

one has

$$[\Delta(\mathbf{p})]_{N,N\pm 2} = \frac{1}{2}\Delta_0 \tag{19}$$

The broken gap symmetry ensures that the diagonal elements are zero. Since $N \approx p_F^2/2m\omega_c \gg 1$ one has for arbitrary gap symmetry

$$\Delta_{N,N'} = \Delta_{N-N'} \tag{20}$$

Then, (8) becomes

$$\sum_{N'} \begin{pmatrix} \left[z - \omega_c\left(N + \frac{1}{2}\right) + \mu\right]\delta_{N',0} & \Delta_{N'} \\ -\Delta_{N'} & \left[-z - \omega_c\left(N + \frac{1}{2}\right) + \mu\right]\delta_{N',0} \end{pmatrix} \hat{G}_{N-N',\bar{N}}(z)$$

$$= (\hat{1})_{N,\bar{N}} \tag{8'}$$

It is convenient to write

$$\mu = \omega_c N_0 + \bar{\mu} \tag{21}$$

where N_0 is the integer part of μ/ω_c then one may rewrite (8') in the new representation

$$f(\varphi) = \sum_n \mathrm{e}^{-in\varphi} f_n, \quad f_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{in\varphi} f(\varphi) d\varphi \tag{22}$$

with $n = N - N_0$. Therefore one finds

$$\hat{G}(z, \varphi, \varphi') = \sum_\lambda \frac{1}{z - E_\lambda} \begin{pmatrix} u_\lambda(\varphi) \\ v_\lambda(\varphi) \end{pmatrix} \otimes (u_\lambda^*(\varphi'), v_\lambda^*(\varphi')) \tag{23}$$

From (21) we have the periodicity $f(\varphi + 2\pi) = f(\varphi)$ as boundary conditions. The matrix $\hat{G}$ can be written in terms of the eigenfunctions $[u_\lambda(\varphi), v_\lambda(\varphi)]$ as

$$\hat{G}(z, \varphi, \varphi') = \sum_\lambda \frac{1}{z - E_\lambda} \begin{pmatrix} u_\lambda(\varphi) \\ v_\lambda(\varphi) \end{pmatrix} \otimes (u_\lambda^*(\varphi'), v_\lambda^*(\varphi')) \tag{24}$$

E_λ is the eigenvalue relative to the chemical potential. u and v satisfy

$$\begin{aligned} \left(E - i\omega_c \tfrac{d}{d\varphi} + \bar{\mu}\right) u(\varphi) + \Delta(\varphi) v(\varphi) = 0 \\ \left(E + i\omega_c \tfrac{d}{d\varphi} - \bar{\mu}\right) v(\varphi) + \Delta(\varphi) u(\varphi) = 0 \end{aligned} \tag{25}$$

The degeneracy of each state per spin is the same as for free electrons in the field $\bar{B}$

$$Ve\bar{B} dp_z/(2\pi)^2 c, \tag{26}$$

were V is the volume. Eqn. (6) may be written as

$$M = -\frac{2\mu e}{2\pi c} \sum_\lambda \overline{|u_\lambda(\varphi)|^2} n(E_\lambda) \tag{6'}$$

The sum runs over all values of E_λ. The spin energy is small compared to other energies in the problem.

It is convenient to remove $\bar{\mu}$ by

$$(u, v) \to \mathrm{e}^{-i(\bar{\mu}/\omega_c)\varphi}(\bar{u}, \bar{v}) \tag{26'}$$

and use functions y and z

$$\bar{u} = \frac{1}{2}(y - iz) \quad \bar{v} = \frac{1}{2}(z - iy) \tag{27}$$

Then, the problem reduces to Schrödinger's equation

$$E^2 y = -\omega_c^2 y'' + [\Delta^2(\varphi) - \omega_c \Delta'(\varphi)]y \tag{28}$$

$$z = -\frac{1}{E}[\omega_c y' + \Delta(\varphi) y] \tag{29}$$

These describe the motion of the quasiparticle along the Fermi surface. In the case of a d-wave gap, the particle is confined to four potential wells formed by $\Delta^2(\varphi)$ around the points $\varphi_k = \frac{\pi}{4} + k\frac{\pi}{2}$. The low energy part ($E \ll \Delta$) of the spectrum has two branches

$$E_n = \pm 2\sqrt{\omega_c \Delta_0 n} \quad n = 0, 1, \ldots \tag{30}$$

When compared to the $\bar{Q}$ terms of (11), (29) is not applicable at too low energy E_n because $(\omega_c \Delta_0)^{1/2}$ is small compared to the Q terms in (11)

$$(d/v_F)(\omega_c \Delta_0)^{1/2} \sim (p_F \xi_0)^{-1/2} \ll 1 \tag{31}$$

To include the Q terms into (22) and (24) we employ a quasi-classical method developed in [13]. The equation can be obtained from (22) and (24) by the substitution

$$E \Rightarrow E + h(\varphi) \equiv E + (e/c)\mathbf{v}_F \cdot \mathbf{Q}(\mathbf{r}) \tag{32}$$

The motion of φ is given by the classical motion of an electron on a circular orbit with Larmour radius $r_L = v_F/\omega_c$

$$\hat{\mathbf{r}} \to \mathbf{r}_0 + (-r_L \cos\omega_c t, r_L \sin\omega_c t) \tag{33}$$

and $v_F = \dot{r}$, $\varphi = \omega_c t$. Having solved for the motion in the periodic potentials $\Delta(\varphi)$ and $h(\varphi)$, the Green's function must be averaged over all trajectories.

Physically, $h(\varphi)$ represents the interaction of the quasiparticle with the currents as the electron moves around the circle of radius r_L with Fermi velocity v_F. The characteristic frequency of the motion through the vortex lattice is $v_F/d \gg \omega_c$. These rapid oscillations while small compared to the gap will smear the spectrum (29). These effects occur with the change of φ of order $\delta\varphi \sim d/r_L \ll 1$.

By applying Poisson's summation formula to (6') taking into account the narrow region about the Fermi surface contributing to the dHvA effect $|\bar{\lambda}| = |\lambda - N_0| \ll N_0$, after integration by parts one finds

$$M_{osc} = \frac{i\mu e}{(2\pi)^2 c}\left(2\Re\left\{\sum_{k,\sigma}\frac{1}{k}\exp\left[2\pi ik\left(\frac{\mu}{\omega_c}\right)\right]\right.\right.$$
$$\left.\left.\int_{-\infty}^{+\infty} e^{2\pi ik\bar{\lambda}}\frac{d}{d\bar{\lambda}}\left[\overline{|u_{\bar{\lambda}}|^2} n(E_{\bar{\lambda}})\right] d\bar{\lambda}\right\}\right] \tag{34}$$

The two terms arising from the differentiation represent oscillations due to level crossings at the Fermi surface

$$\Rightarrow \int_{-\infty}^{+\infty} e^{2\pi ik\bar{\lambda}}\overline{|u_\lambda|^2}\delta(E_{\bar{\lambda}})d\bar{\lambda} \tag{33'}$$

or the effect of the variation of the eigenfunction

$$\Rightarrow \int_{-\infty}^{+\infty} e^{2\pi ik\bar{\lambda}}\frac{d}{d\bar{\lambda}}\left(\overline{|u_{\bar{\lambda}}|^2}\right) d\bar{\lambda} \tag{33''}$$

To apply this scheme to the two situations discussed above we first treat a line of zeros of the gap with the field direction such that the extremal orbit lies on it. Since the gap is zero the electron wave function is of the form

$$\exp(\pm iS(\varphi)) \quad S(\varphi) = \frac{1}{\omega_c}\int_0^{\varphi}\left[E + h(\varphi')\right] d\varphi' \tag{35}$$

From periodicity, the eigenvalue problem becomes

$$2\pi\bar{\lambda} = S(2\pi) = \frac{1}{\omega_c}\int_0^{2\pi}\left[E_\lambda + h(\varphi')\right] d\varphi' \tag{34'}$$

By comparing (33) and (33'), one finds

$$M^S_{osc}(k) = M^N_{osc}(k)\left\langle \exp\frac{ik}{\omega_c}\int_0^{2\pi} h(\varphi')d\varphi'\right\rangle \tag{36}$$

The average is taken over all orbits. By combining (3), (31) and (32), the exponent may be expressed as an integral over orbit area

$$\frac{1}{\omega_c}\int_0^{2\pi} h(\varphi)d\varphi = \frac{e}{c}\oint \mathbf{Q}\cdot d\mathbf{l} =$$
$$(1/2\phi_0)\int\int\left[\bar{B}-\phi_0\sum_i \delta(\mathbf{r}-\mathbf{r}_i)\right]dS \tag{37}$$

Thus the exponent in (35) measures the fluctuations in the number of fluxes encircled by the electron orbits n_φ. For a periodic vortex lattice (36) is evaluated with a Gaussian distribution for n_φ with a mean value $\bar{n}_\varphi$ given by

$$\overline{n_\varphi} = (2\pi r_L d)/d^2 = 2\pi r_L/d \tag{38}$$

For the Dingle factor $\exp[-2\pi cm/e\tau\bar{B}]$ one finds

$$1/\tau \sim \Delta_0(\xi_0/d) \tag{39}$$

For an s-wave gap $1/\tau = \Delta_0$ so that the dHvA amplitude is enhanced exponentially by the factor ξ_0/d.

For the case of a d-wave gap, as we have stated (29) is not accurate for small n due to the vortex currents. Nevertheless, the structure of E_n shows that in contrast to the normal state where levels move through the chemical potential here levels move symmetrically away from the Fermi surface, indicating a small dHvA amplitude in this case. The existence of levels in the vicinity of $E = 0$ is a topological property for a soliton like problem [14] at each point $\varphi_k = \frac{\pi}{4} + k\frac{\pi}{2}$ where $\Delta(\varphi)$ changes sign.

In conclusion we have presented a method for treating electron motion in a superconductor in the presence of a field $B \ll H_{C2}$. For a line of zeros, the dHvA amplitude is strongly enhanced relative to an s-wave gap. For point zeros, the effect is expected to be weak. This paper is based on the work of L.P. Gor'kov and J.R. Schrieffer (15). This work was supported by the NHMFL through NSF grant DMR 0084173 and the State of Florida.

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